Variational Segmentation and Contour Matching of Non-Rigid Moving Object

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Abstract. In this paper we propose a method for variational segmentation and contour matching of non-rigid objects in image sequences which can deal with the occlusions. The method is based on a region-based active contour model of the Chan-Vese, augmented with a frame-to-frame interaction term which uses the segmentation result from the previous frame as a shape prior. This method has given good results despite the presence of minor occlusions, but can not handle significant occlusions. We have extended this approach by adding a registration step between two consecutive contours. This registration step is based on a novel variational formulation and gives also a mapping of the intensities from the interior of the previous contour to the next. With this information occlusions can be detected from deviations from predicted intensities and the missing intensities in the occluded areas can then be reconstructed. The performance of the method is shown with experiments on synthetic and real image sequences.

1 Introduction

Segmentation is an important and difficult process in computer vision, with the purpose of dividing a given image into one or several meaningful regions or objects. This process is more difficult when the objects to be segmented are moving and non-rigid and even more when there are severe occlusions. The shape of non-rigid, moving objects may vary a lot along image sequences due to, for instance, deformations or occlusions, which puts additional constraints on the segmentation process. In particular we would like to distinguish *real* shape deformations of the object from *apparent* shape deformations due to occlusions.

There have been a number of methods proposed and applied to this problem. Active contours are powerful methods for image segmentation; either boundary-based such as geodesic active contours [1], or region-based such as Chan-Vese models [2], which are formulated as variational problems. Those variational formulations perform quite well and have often been applied based on level sets. Active contour based segmentation methods often fail due to noise, clutter and occlusion. In order to make the segmentation process robust against these effects, shape priors have been proposed to be incorporated into the segmentation process. In recent years, many researchers have successfully introduced shape priors into segmentation methods such as in [3–9].

We are interested in segmenting non-rigid moving objects in image sequences. When the objects are non-rigid, an appropriate segmentation method that can deal with shape deformations should be used. The application of active contour methods for segmentation in image sequences gives promising results as in [10–12]. These methods use variants of the classical Chan-Vese model as the basis for segmentation. In [10], for instance, it is proposed to simply use the result from one image as an initializer in the segmentation of the next.

Another major problem for segmentation methods for image sequences is the presence of occlusions. Minor occlusions can usually be handled by some kind of shape prior. However, major occlusions is still a big problem. In order to improve the robustness of the segmentation methods in the presence of occlusions, it is necessary to detect the occlusions. The occluded area can then either be excluded from segmentation process or reconstructed [13–15].

The main purpose of this paper is to propose and analyze a novel variational segmentation method for image sequences, that can both deal with shape deformations and at the same time is robust to noise, clutter and occlusions. The proposed method is based on minimizing an energy functional containing the standard Chan-Vese functional as one part and a term that penalizes the deviation from the previous shape as a second part. The second part of the functional is based on a transformed distance map to the previous contour, where different transformation groups, such as Euclidean, similarity or affine, can be used depending on the particular application. This variational framework is then augmented with a novel contour flow algorithm, giving a mapping of the intensities inside the contour of one image to the inside of the contour in the next image. Using this mapping, occlusions can be detected by simply thresholding the difference between the transformed intensities and the observed ones in the novel image.

This paper is organized as follows: in Sect. 2 we discuss the proposed segmentation of image sequences. The variational contour matching is described in Sect. 3 and how this can be used to detect and locate the occlusion is described in Sect. 4. Experimental results of the model are presented in Sect. 5 and we end the paper with some conclusions.

2 Segmentation of Image Sequences

In this section, we describe the region-based segmentation model of Chan-Vese [2] and a variational model for updating segmentation results from one frame to the next in an image sequence.

2.1 Region-Based Segmentation

The idea of the Chan-Vese model [2] is to find a contour Γ such that the image I is optimally approximated by a gray scale value μ_{int} on $\text{int}(\Gamma)$, the *inside* of Γ, and by another gray scale value μ_{ext} on $\text{ext}(\Gamma)$, the *outside* of Γ. The optimal contour Γ^* is defined as the solution of the variational problem,

$$E_{CV}(\Gamma^*) = \min_{\Gamma} E_{CV}(\Gamma), \tag{1}$$

where E_{CV} is the Chan-Vese functional,

$$E_{CV}(\Gamma) = \alpha|\Gamma| + \beta\Big\{\frac{1}{2}\int_{\text{int}(\Gamma)} (I(\mathbf{x}) - \mu_{\text{int}})^2\, d\mathbf{x} + \frac{1}{2}\int_{\text{ext}(\Gamma)} (I(\mathbf{x}) - \mu_{\text{ext}})^2\, d\mathbf{x}\Big\}. \tag{2}$$

Here $|\Gamma|$ is the arc length of the contour, $\alpha, \beta > 0$ are weight parameters, and

$$\mu_{\text{int}} = \mu_{\text{int}}(\Gamma) = \frac{1}{|\,\text{int}(\Gamma)|}\int_{\text{int}(\Gamma)} I(\mathbf{x})\, d\mathbf{x}, \tag{3}$$

$$\mu_{\text{ext}} = \mu_{\text{ext}}(\Gamma) = \frac{1}{|\,\text{ext}(\Gamma)|}\int_{\text{ext}(\Gamma)} I(\mathbf{x})\, d\mathbf{x}. \tag{4}$$

The gradient descent flow for the problem of minimizing a functional $E_{CV}(\Gamma)$ is the solution to initial value problem:

$$\frac{d}{dt}\Gamma(t) = -\nabla E_{CV}(\Gamma(t)), \qquad \Gamma(0) = \Gamma_0, \tag{5}$$

where Γ_0 is an initial contour. Here $\nabla E_{CV}(\Gamma)$ is the L^2-gradient of the energy functional $E_{CV}(\Gamma)$, cf. e.g. [16] for definitions of these notions. Then the L^2-gradient of E_{CV} is

$$\nabla E_{CV}(\Gamma) = \alpha\kappa + \beta\big[\frac{1}{2}(I - \mu_{\text{int}}(\Gamma))^2 - \frac{1}{2}(I - \mu_{\text{ext}}(\Gamma))^2\big], \tag{6}$$

where κ is the curvature.

In the level set framework [17], a curve evolution, $t \mapsto \Gamma(t)$, can be represented by a time dependent level set function $\phi : \mathbf{R}^2 \times \mathbf{R} \to \mathbf{R}$ as $\Gamma(t) = \{\mathbf{x} \in \mathbf{R}^2\ ;\ \phi(\mathbf{x}, t) = 0\}$, $\phi(\mathbf{x}) < 0$ and $\phi(\mathbf{x}) > 0$ are the regions inside and the outside of Γ, respectively. The normal velocity of $t \mapsto \Gamma(t)$ is the scalar function $d\Gamma/dt$ defined by

$$\frac{d}{dt}\Gamma(t)(\mathbf{x}) := -\frac{\partial\phi(\mathbf{x}, t)/\partial t}{|\nabla\phi(\mathbf{x}, t)|} \qquad (\mathbf{x} \in \Gamma(t)). \tag{7}$$

Recall that the outward unit normal $\mathbf{n}$ and the curvature κ can be expressed in terms of ϕ as $\mathbf{n} = \nabla\phi/|\nabla\phi|$ and $\kappa = \nabla \cdot \big(\nabla\phi/|\nabla\phi|\big)$.

Combined with the definition of gradient descent evolutions (5) and the formula for the normal velocity (7) this gives the gradient descent procedure in the level set framework:

$$\frac{\partial\phi}{\partial t} = \Big(\alpha\kappa + \beta\big[\frac{1}{2}(I - \mu_{\text{int}}(\Gamma))^2 - \frac{1}{2}(I - \mu_{\text{ext}}(\Gamma))^2\big]\Big)|\nabla\phi|,$$

where $\phi(\mathbf{x}, 0) = \phi_0(\mathbf{x})$ represents the initial contour Γ_0.

2.2 The Interaction Term

The interaction $E_I(\Gamma_0, \Gamma)$ between a fixed contour Γ_0 and an active contour Γ may be regarded as a shape prior and be chosen in several different ways, such as the pseudo-distances, cf. [6], and the area of the symmetric difference of the sets $\mathrm{int}(\Gamma)$ and $\mathrm{int}(\Gamma_0)$, cf. [3].

Let $\phi_0 : D \to \mathbf{R}$ denotes the *signed distance function* associated with the contour Γ_0 and $\mathbf{a} \in \mathbf{R}^2$ is a group of translations. We want to determine the optimal translation vector $\mathbf{a} = \mathbf{a}(\Gamma)$, then the interaction $E_I = E_I(\Gamma_0, \Gamma)$ is defined by the formula,

$$E_I(\Gamma_0, \Gamma) = \min_{\mathbf{a}} \int_{\mathrm{int}(\Gamma)} \phi_0(\mathbf{x} - \mathbf{a})\, d\mathbf{x}. \tag{8}$$

Minimizing over groups of transformations is the standard devise to obtain pose-invariant interactions, see [3] and [6].

Since this is an optimization problem $\mathbf{a}(\Gamma)$ can be found using the gradient descent procedure. The optimal translation $\mathbf{a}(\Gamma)$ can then be obtained as the limit, as time t tends to infinity, of the solution to initial value problem

$$\dot{\mathbf{a}}(t) = \int_{\mathrm{int}(\Gamma)} \nabla\phi_0(\mathbf{x} - \mathbf{a}(t))\, d\mathbf{x}, \qquad \mathbf{a}(0) = 0. \tag{9}$$

Similar gradient descent schemes can be devised for rotations and scalings (in the case of similarity transforms), cf. [3].

2.3 Using the Interaction Term in Segmentation of Image Sequences

Let $I_j : D \to \mathbf{R}$, $j = 1, \ldots, N$, be a succession of N frames from a given image sequence. Also, for some integer k, $1 \le k \le N$, suppose that all the frames $I_1, I_2, \ldots, I_{k-1}$ have already been segmented, such that the corresponding contours $\Gamma_1, \Gamma_2, \ldots, \Gamma_{k-1}$ are available. In order to take advantage of the prior knowledge obtained from earlier frames in the segmentation of I_k, we propose the following method: If $k = 1$, i.e. if no previous frames have actually been segmented, then we just use the standard Chan-Vese model, as presented in Sect. 2.1. If $k > 1$, then the segmentation of I_k is given by the contour Γ_k which minimizes an *augmented* Chan-Vese functional of the form,

$$E^A_{CV}(\Gamma_{k-1}, \Gamma_k) := E_{CV}(\Gamma_k) + \gamma E_I(\Gamma_{k-1}, \Gamma_k), \tag{10}$$

where E_{CV} is the Chan-Vese functional, $E_I = E_I(\Gamma_{k-1}, \Gamma_k)$ is an *interaction term*, which penalizes deviations of the current active contour Γ_k from the previous one, Γ_{k-1}, and $\gamma > 0$ is a coupling constant which determines the strength of the interaction.

The augmented Chan-Vese functional (10) is minimized using standard gradient descent (5) described in Sect. 2.1 with ∇E equal to

$$\nabla E^A_{CV}(\Gamma_{k-1}, \Gamma_k) := \nabla E_{CV}(\Gamma_k) + \gamma \nabla E_I(\Gamma_{k-1}; \Gamma_k), \tag{11}$$

and the initial contour $\Gamma(0) = \Gamma_{k-1}$. Here ∇E_{CV} is the L^2-gradient (6) of the Chan-Vese functional, and ∇E_I the L^2-gradient of the interaction term, which is given by the formula,

$$\nabla E_I(\Gamma_{k-1}, \Gamma_k; \mathbf{x}) = \phi_{k-1}(\mathbf{x} - \mathbf{a}(\Gamma_k)), \qquad (\text{for } \mathbf{x} \in \Gamma_k). \tag{12}$$

Here ϕ_{k-1} is the signed distance function for Γ_{k-1}.

We use the Chan-Vese model to segment a selected object with approximately uniform intensity and apply the proposed method frame-by-frame. First we compute the optimal translation vector (9) based on the previous contour, we then use this vector to translate the previous contour until it is aligned to the optimal position (12). Then the minimum of the functional (10) is obtained by the gradient descent procedure (11) implemented in the level set framework outline in Sect. 2. This procedure is iterated until it converges.

3 A Contour Matching Problem

In this section we are going to present a variational solution to the following contour matching problem: Suppose we have two simple closed curves Γ_1 and Γ_2 contained in the image domain Ω. Find the "most economical" mapping $\Phi = \Phi(x) : \Omega \to \mathbf{R}^2$ such that Φ maps Γ_1 onto Γ_2, i.e. $\phi(\Gamma_1) = \Gamma_2$. The latter condition is to be understood in the sense that if $\alpha = \alpha(s) : [0,1] \to \Omega$ is a positively oriented parametrization of S_1, then $\beta(s) = \Phi(\alpha(s)) : [0,1] \to \Omega$ is a positively oriented parametrization of Γ_2 (allowing some parts of Γ_2 to be covered multiple times).

To present our variational solution of this problem, let $\mathcal{M}$ denote the set of twice differential mappings Φ which maps Γ_1 to Γ_2 in the above sense. Loosely speaking

$$\mathcal{M} = \{\Phi \in C^2(\Omega; \mathbf{R}^2) \,|\, \Phi(\Gamma_1) = \Gamma_2\}.$$

Moreover, given a mapping $\Phi : \Omega \to \mathbf{R}^2$, not necessarily a member of $\mathcal{M}$, then we express Φ in the form $\Phi(x) = x + U(x)$, where the vector valued function $U = U(x) : \Omega \to \mathbf{R}^2$ is called the *displacement field associated with* Φ, or simply the displacement field. It is sometimes necessary to write out the components of the displacement field; $U(x) = (u_1(x), u_2(x))^T$.

We now define the "most economical" map to be the member Φ^* of $\mathcal{M}$ which minimizes the following energy functional:

$$E[\Phi] = \frac{1}{2} \int_\Omega \|DU(x)\|_F^2 \, d\mathbf{x}, \tag{13}$$

where $\|DU(x)\|_F$ denotes the Frobenius norm of $DU(x) = [\nabla u_1(x), \nabla u_2(x)]^T$, which for an arbitrary matrix $A \in \mathbf{R}^{2\times 2}$ is defined by $\|A\|_F^2 = \mathrm{tr}(A^T A)$. That is, the optimal matching is given by

$$\Phi^* = \underset{\Phi \in \mathcal{M}}{\arg\min}\, E[\Phi]. \tag{14}$$

Using that $E[\Phi]$ can be written in the form

$$E[\Phi] = \frac{1}{2}\int_{\Omega} |\nabla u_1(x)|^2 + |\nabla u_2(x)|^2 \, d\mathbf{x}, \tag{15}$$

it is easy to see that the Gâteaux derivative of $E[\Phi]$ is given by

$$\begin{aligned} dE[\Phi; V] &= \int_{\Omega} \nabla u_1(x) \cdot \nabla v_1(x) + \nabla u_2(x) \cdot \nabla v_2(x) \, d\mathbf{x} \\ &= \int_{\Omega} \mathrm{tr}(DU(x)^T DV(x)) \, d\mathbf{x}, \end{aligned}$$

for any displacement field $V(x) = (v_1(x), v_2(x))^T$. After integration by parts we find that the necessary condition for $\Phi^*(x) = x + U^*(x)$ to be a solution of the minimization problem (14) takes the form

$$0 = -\int_{\Omega} \Delta U^*(x) \cdot V(x) \, d\mathbf{x}, \tag{16}$$

for any *admissible* displacement field variation $V = V(x)$. Here $\Delta U^*(x) = (\Delta u_1(x), \Delta u_2(x))^T$ is the Laplacian of the vector valued function $U^* = U^*(x)$. Since every admissible mapping Φ must map the initial contour Γ_1 onto the target contour Γ_2, it can be shown that any displacement field variation V must satisfy

$$V(x) \cdot \mathbf{n}_{S_2}(x + U^*(X)) = 0 \quad \text{for all } x \in \Gamma_1. \tag{17}$$

Notice that this condition only has to be satisfied precisely on the curve Γ_1, and that $V = V(x)$ is allowed to vary freely away from the initial contour. The interpretation of the above condition is that the displacement field variation at $x \in \Gamma_1$ must be tangent to the target contour Γ_2 at the point $y = \Phi(x)$. In view of this interpretation of (17) it is not difficult to see that necessary condition (16) implies that the solution Φ^* of the minimization problem (14) must satisfy the following Euler-Lagrange equation:

$$0 = \begin{cases} \Delta U^* - (\Delta U^* \cdot \mathbf{n}^*_{\Gamma_2})\, \mathbf{n}^*_{\Gamma_2}, & \text{on } \Gamma_1, \\ \Delta U^*, & \text{otherwise}, \end{cases} \tag{18}$$

where $\mathbf{n}^*_{\Gamma_2}(x) = \mathbf{n}_{\Gamma_2}(x + U^*(x))$, $x \in \Gamma_1$, is the pullback of the normal field of the target contour Γ_2 to the initial contour Γ_1. The standard way of solving (18) is to use the gradient descent method: Let $U = U(t, x)$ be the time-dependent displacement field which solves the evolution PDE

$$\frac{\partial U}{\partial t} = \begin{cases} \Delta U - (\Delta U \cdot \mathbf{n}^*_{\Gamma_2})\, \mathbf{n}^*_{\Gamma_2}, & \text{on } \Gamma_1, \\ \Delta U, & \text{otherwise}, \end{cases} \tag{19}$$

where the initial displacement $U(0, x) = U_0(x) \in \mathcal{M}$ specified by the user, and $U = 0$ on $\partial\Omega$, the boundary of Ω (Dirichlet boundary condition). Then $U^*(x) = \lim_{t\to\infty} U(t, x)$ is a solution of the Euler-Lagrange equation (18).

Notice that the PDE (19) coincides with the so-called *geometry-constrained diffusion* introduced in [18]. Thus we have incidentally found a variational formulation of the non-rigid registration problem considered there.

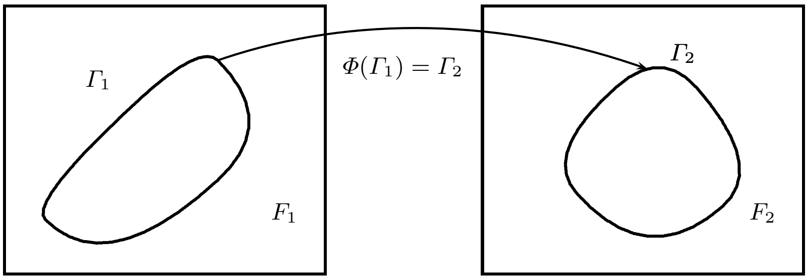

Fig. 1. Given two closed curves Γ_1 and Γ_2 contained in two images F_1 and F_2, Φ maps F_1 onto F_2 such that Γ_1 is mapped onto Γ_2 (i.e. $\Phi(\Gamma_1) = \Gamma_2$).

4 Detect and Locate the Occlusion

The mapping $\Phi = \Phi(x) : \Omega \to \mathbf{R}^2$ such that Φ maps Γ_1 onto Γ_2 is an estimation of the displacement (motion and deformation) of the boundary of an object between two frames. By finding the displacement of the contour, a consistent displacement of the intensities inside the closed curve Γ_1 can also be found. Φ maps Γ_1 onto Γ_2 and pixels inside Γ_1 are mapped inside Γ_2. This displacement field which only depends on displacement - or registration - of the contour (and not on the image intensities) can then be used to map the intensities inside Γ_1 into Γ_2. After mapping, the intensities inside Γ_1 and Γ_2 can be compared and then be classified as the same or different value. Since we can still find the contour in the occluded area, therefore we can also compute the displacement field even in the occluded area.

After the occlusion has been detected, the segmentation can be further improved by again employing the previously described Chan-Vese-method augmented with an interaction term. However, in this second stage, the integration is only performed over the area of the image where no occlusion has been detected. This procedure treats the occluded area in the same way as a part of the image with missing data as in [19], which is reasonable .

5 Experiments

5.1 Segmentation

In this section we present the results obtained from experiment using synthetic image sequence. We use the Chan-Vese model to segment a selected object with approximately uniform intensity and apply the proposed method frame-by-frame. The minimization of the functional is obtained by the gradient descent procedure (11) implemented in the level set framework. See also [17].

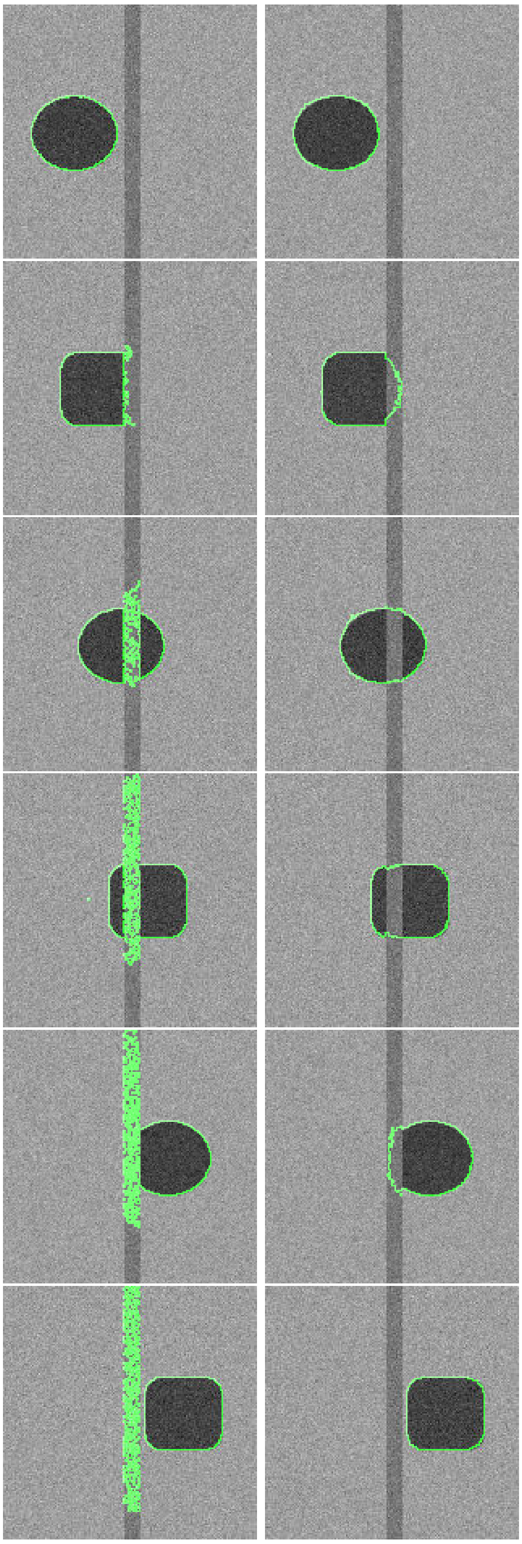

Fig. 2. Segmentation of a non-rigid object in a synthetic image sequences with additive Gaussian noise (Frame 1-7). Without the interaction term, noise in the occlusion is captured (Left column). This is avoided when the interaction term is included (Right column).

The classical Chan-Vese method will have problems segmenting an object if occlusions appear in the image which cover the whole or parts of the selected object. In Fig. 2 and Fig. 5, we show the segmentation results for a non-rigid object in a synthetic image sequence and for a walking human in a real image sequence (available at http://homepages.inf. ed.ac.uk/rbf/CAVIAR/), respectively, where occlusions occur. The classical Chan-Vese method fails to segment the selected object when it reaches the occlusion (Left column). Using the proposed method, which uses the frame-to-frame interaction term, we obtain much better results (Right column).

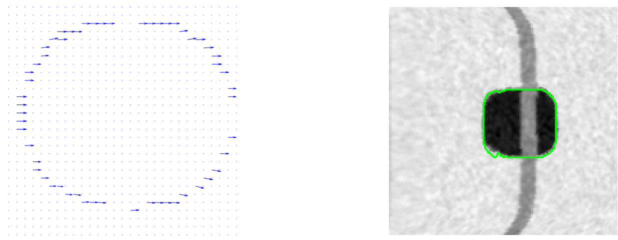

Fig. 3. Left: Deformation field. Right: Frame 4 after deformation according to the displacement field onto Frame 5.

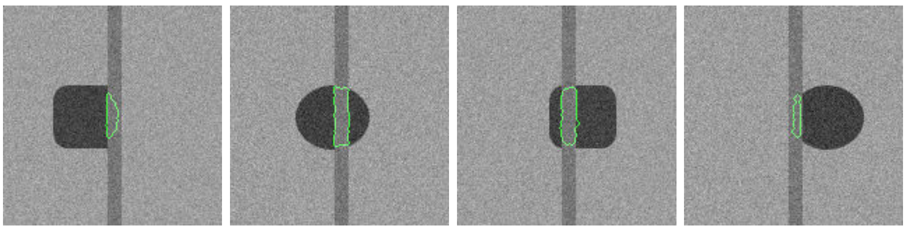

Fig. 4. The occluded regions of the Frame 3-6 of Fig. 2 can be detected and located

In both experiments the coupling constant γ is varied to see the influence of the interaction term on the segmentation results. The contour is only slightly affected by the prior if γ is small. On the other hand, if γ is too large, the contour will be close to a similarity transformed version of the prior.

5.2 Contour Matching and Occlusion Detection

As described in Sect. 3 and Sect. 4, occlusion can be detected and located by deforming the current frame according to the displacement and compare the

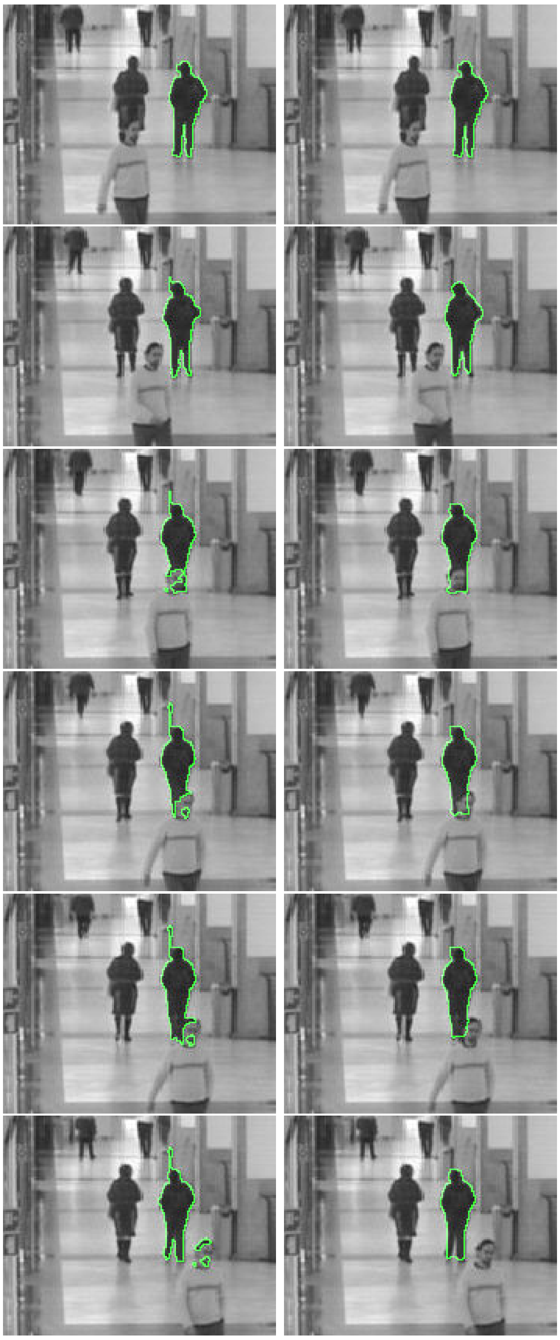

Fig. 5. Segmentation of a person covered by an occlusion in the human walking sequence. Left column: without interaction term, and Right column: with interaction term

deformed frame with the next frame (inside the contour Γ_2). First we compute the displacement field based on the segmentation results of two frames. In Fig. 3, we show the displacement field of Frame 4 and 5. With this displacement field, we can do full deformation of the Frame 4 onto Frame 5 (Fig. 3 right) and then compare the intensities between Frame 5 and deformed Frame 4. By comparing, we can then classify the intensities as having the same or different value by thresholding. The results for the synthetic image sequence are presented in Fig. 4 and for the human walking sequence in Fig. 6.

Fig. 6. The occluded regions of the Frame 3 and 4 of Fig. 5 are detected and located by predicting the intensities inside the contour of the walking person.

6 Conclusions

We have presented a new method for segmentation and contour matching of image sequences containing nonrigid, moving objects, that also can handle occlusions. The proposed segmentation method is formulated as variational problem, with one part of the functional corresponding to the Chan-Vese model and another part corresponding to the pose-invariant interaction with a shape prior based on the previous contour. The optimal transformation as well as the shape deformation are determined by minimization of an energy functional using a gradient descent scheme. This segmentation method is augmented with a contour flow estimation algorithm based on a novel variational formulation. The estimated contour flow makes it possible to extract occluded areas and then further refine the segmentation. Preliminary results are shown and its performance looks promising both in terms of segmentation and occlusion detection.

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