580.222 SYSTEMS AND CONTROLS

SPRING 2007- Homework Set 1
Distributed: March 21st 2007
Due: March 28th 2007

READING ASSIGNMENT

Chapter 3.1 and Appendix A of Franklin. Chapter 9.1-9.6 and Appendix A of Oppenheim

PROBLEMS TO BE GRADED

Problem 1: Franklin 3.4 (b)-(c) and 3.5 (c)-(e):
Find the Laplace transform **and region of convergence** of the following time functions ($*$ denotes convolution):
(b) $f(t) = t\cos 3t$
(c) $f(t) = te^{-t} + 2t\cos t$
(c) $f(t) = (\sin t)/t$
(d) $f(t) = \sin t * \sin t$
(e) $f(t) = \int_0^\infty \cos(t-\tau)\sin\tau d\tau$

Problem 2: Franklin 3.7:
Find the time function corresponding to each of the following Laplace transforms using partial fraction expansions:
(b) $F(s) = \frac{10}{s(s+1)(s+10)}$
(d) $F(s) = \frac{3s^2+9s+12}{(s+2)(s^2+5s+11)}$
(h) $F(s) = \frac{1}{s^6}$
(i) $F(s) = \frac{4}{s^4+4}$
(j) $F(s) = \frac{e^{-s}}{s^2}$

Problem 3: Franklin 3.8:
Find the time function corresponding to each of the following Laplace transforms:
(a) $F(s) = \frac{1}{s(s+2)^2}$
(c) $F(s) = \frac{2(s^2+s+1)}{s(s+1)^2}$
(e) $F(s) = \frac{2(s+2)(s+5)^2}{(s+1)(s^2+4)^2}$
(f) $F(s) = \frac{(s^2-1)}{(s^2+1)^2}$
(g) $F(s) = \tan^{-1}(\frac{1}{s})$

Problem 4: Franklin 3.9:
Solve the following ordinary differential equations using Laplace transforms:
(b) $\ddot{y}(t) - 2\dot{y}(t) + 4y(t) = 0; y(0) = 1; \dot{y}(0) = 2$
(d) $\ddot{y}(t) + 3y(t) = \sin t; y(0) = 1; \dot{y}(0) = 2$
(f) $\ddot{y}(t) + y(t) = t; y(0) = 1; \dot{y}(0) = -1$

Problem 5:
Consider a matrix $A \in \mathbb{R}^{n\times n}$ and define the matrix function

$$X(t) = \exp(At) = \sum_{n=0}^{\infty} \frac{(At)^n}{n!}$$

a) Show that $X(t)$ satisfies the differential equation $\dot{X}(t) = AX(t)$, $X(0) = I$.

b) Show that the Laplace transform of $\exp(At)$ is $(sI - A)^{-1}$.
c) Use the inverse Laplace transform to compute $\exp(At)$ for

$$A = \begin{bmatrix} 0 & w \\ -w & 0 \end{bmatrix}$$