580.222 SYSTEMS AND CONTROLS

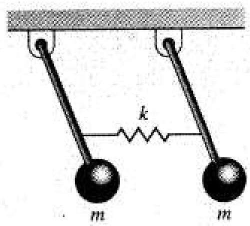
SPRING 2007- Homework Set 2
Distributed: March 28th 2007
Due: Wednesday April 4th 2007

READING ASSIGNMENT
Chapter 2.1, 2.2, 3.2, 3.3, 3.4 of Franklin

PROBLEMS TO BE GRADED

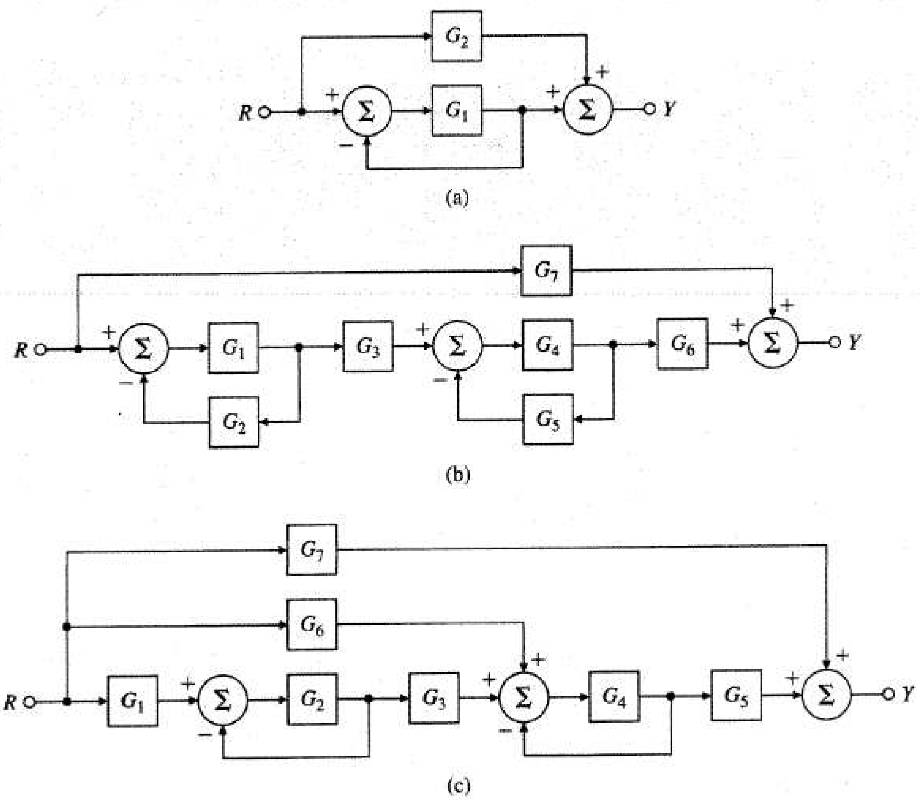
1. Write the equations of motion for the double-pendulum system shown in Fig. 2.35. Assume the displacement angles of the pendulums are small enough to ensure that the spring is always horizontal. The pendulum rods are taken to be massless, of length l, and the springs are attached 3/4 of the way down.

Figure 2.35
Double pendulum



2. Find the transfer functions for the block diagrams in Fig. 3.44.

Figure 3.44
Block diagrams for Problem 3.19



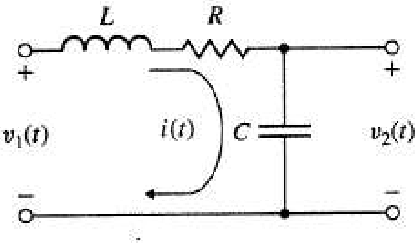
3. For the electric circuit shown in Fig. 3.47, find the following:

(a) The time-domain equation relating $i(t)$ and $v_1(t)$;

(b) The time-domain equation relating $i(t)$ and $v_2(t)$;

(c) Assuming all initial conditions are zero, the transfer function $V_2(s)/V_1(s)$ and the damping ratio ζ and undamped natural frequency ω_n of the system;

Figure 3.47
Circuit for Problem 3.22



(d) The values of R that will result in $v_2(t)$ having an overshoot of no more than 25%, assuming $v_1(t)$ is a unit step, $L = 10mH$, and $C = 4\mu F$.

4. *Modeling glucose metabolism*

Glucose metabolism can be represented by the two compartment model shown below. Glucose enters gastrointestinal (GI) tract and gets absorbed into the bloodstream where it is metabolized. A rabbit is fed with glucose. Let the concentration of glucose in the GI tract and bloodstream at time t be $x_1(t)$ and $x_2(t)$ respectively, and let the rate of glucose ingestion be $z(t)$. Assume the volume of the two compartments are both 1 liter. Also assume that there is a negligible amount of glucose in the bloodstream and GI tract before the oral dose.

$$\begin{array}{ccc} z(t) & & \\ \downarrow & & \\ \boxed{\text{GI tract}} & \xrightarrow{\lambda_1} & \boxed{\text{Blood}} \\ & & \downarrow \\ & & \lambda_2 \end{array}$$

(a) Let us assume glucose is absorbed into the blood stream at a rate in proportion to $x_1(t)$ with a proportionality constant λ_1. Meanwhile, glucose is metabolized at a rate in proportion to $x_2(t)$ with proportionality constant λ_2. Set up the differential equations for $x_1(t)$ and $x_2(t)$.

(b) In this system, $x_1(t)$ and $x_2(t)$ are the internal states. Now let $x_2(t)$ be our readout. Then the system output is $y(t) = x_2(t)$. Find the transfer function of this system. (Hint: are the solutions the same for $\lambda_1 = \lambda_2$ and $\lambda_1 \neq \lambda_2$?)

(c) Now let us only consider the case $\lambda_1 \neq \lambda_2$. What is the output $y(t)$ if the input is $z(t) = \delta(t)$? When does the blood glucose concentration reach a maximum?

5. *Modeling cellular regulatory pathways*

c-Fos is a transcription factor that is involved in mitotic regulation. Your lab has discovered a class of RTK receptors that can activate c-Fos expression, but are themselves inhibited at high levels of c-Fos. You can activate these receptors by introducing a drug X. Consider a simple model of this process, where x_1 = concentration of c-Fos, x_2 = number of activated RTK receptors and $u(t)$ = concentration of drug X,

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -x_1 + u(t)$$

You are interested in understanding changes in c-Fos concentration in response to stimulation by drug X. Let $y(t)$, the desired output be $x_1(t)$.

(a) Find the transfer function $\frac{Y(s)}{U(s)}$.

(b) What is $x_1(t)$ and $x_2(t)$ when $x_1(0) = x_{10}$ and $x_2(0) = x_{20}$?

(c) Find $y(t)$ in response to a step change in drug X concentration, i.e. $u(t) = 1_+(t)$.

(d) Find $y(t)$ in response to a periodic application of drug X, i.e. $u(t) = \cos(w_0 t) + K1_+(t)$. What happens when $w_0 = 1$?

(e) Using the simulink package in MATLAB, for part d) show

i. Setup of simulink model (sinks, sources, transfer function blocks)

ii. Time course of $y(t)$ for $w_0 = 1, 1.1, 1.25, 1.5, 2$. Take $K = 1, T_{sim} = 10$. Ignore negative values. What can you say about the behaviour of $y(t)$ around $w_0 = 1$?