580.222 SYSTEMS AND CONTROLS

SPRING 2007- Homework Set 3
Distributed: April 4th 2007
Due: April 11th 2007

READING ASSIGNMENT

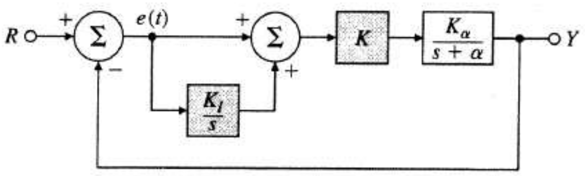
Chapter 3.2, 3.3, 3.4, 3.5 and 3.7 of Franklin

PROBLEMS TO BE GRADED

Franklin 3.26, 3.32, 3.35, Problems 4&5

3.26. Suppose you are to design a unity feedback controller for a first-order plant depicted in Fig. 3.50. (As you will learn in Chapter 4, the configuration shown is referred to as a proportional-integral controller.) You are to design the controller so that the closed-loop poles lie within the shaded regions shown in Fig. 3.51.

Figure 3.50
Unity feedback system for Problem 3.26

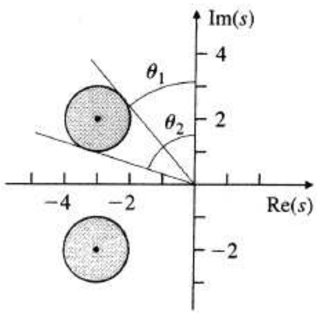


(a) What values of ω_n and ζ correspond to the shaded regions in Fig. 3.51? (A simple estimate from the figure is sufficient.)

(b) Let $K_\alpha = \alpha = 2$. Find values for K and K_I so that the poles of the closed-loop system lie within the shaded regions.

(c) Prove that no matter what the values of K_α and α are, the controller provides enough flexibility to place the poles anywhere in the complex (left-half) plane.

Figure 3.51
Desired closed-loop pole locations for Problem 3.26

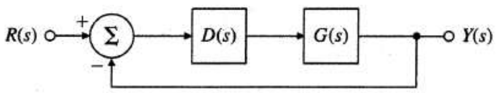


3.32. Consider the system shown in Fig. 3.55, where

$$G(s) = \frac{1}{s(s+3)} \quad \text{and} \quad D(s) = \frac{K(s+z)}{s+p}. \tag{3.82}$$

Find K, z, and p so that the closed-loop system has a 10% overshoot to a step input and a settling time of 1.5 sec (1% criterion).

Figure 3.55
Unity feedback system for Problem 3.32



3.35. Consider the following second-order system with an extra pole:

$$H(s) = \frac{\omega_n^2 p}{(s+p)(s^2 + 2\zeta\omega_n s + \omega_n^2)}.$$

Show that the unit-step response is

$$y(t) = 1 + Ae^{-pt} + Be^{-\sigma t}\sin(\omega_d t - \theta),$$

where

$$A = \frac{-\omega_n^2}{\omega_n^2 - 2\zeta\omega_n p + p^2},$$

$$B = \frac{p}{\sqrt{(p^2 - 2\zeta\omega_n p + \omega_n^2)(1-\zeta^2)}},$$

$$\theta = \tan^{-1}\frac{\sqrt{1-\zeta^2}}{-\zeta} + \tan^{-1}\frac{\omega_n\sqrt{1-\zeta^2}}{p - \zeta\omega_n}.$$

(a) Which term dominates $y(t)$ as p gets large?

(b) Give approximate values for A and B for small values of p.

(c) Which term dominates as p gets small? (Small with respect to what?)

(d) Using the preceding explicit expression for $y(t)$ or the step command in MATLAB, and assuming that $\omega_n = 1$ and $\zeta = 0.7$, plot the step response of the preceding system for several values of p ranging from very small to very large. At what point does the extra pole cease to have much effect on the system response?

Problem 4: Simulink

Refer to Problem 2 (Franklin 3.32). Simulate the system in 3.32 with the following parameters:

a. $z = 3.1, p = 6.3, K = 20.25$

b. $z = 5.77, p = 57.7, K = 222.45$

c. $z = 3.1, K = 222.45, p = -2$

Plot the output $y(t)$ for the above parameters and explain response in each case.

Problem 5: *Modeling neural circuits*

The amygdala is a region of the brain that is associated with emotions such as fear and anxiety. You have noted that in some cases of hereditary schizophrenia, patients express a mutated dopamine receptor in neurons that is refractory to excessive firing, i.e. the more frequently a neuron fires, the less sensitive the receptor is to dopamine, which reduces the rate of subsequent firing.

Let $y(t)$ be the number of times a neuron fires. $\dot{y}(t)$ is the firing rate of the neuron. $\ddot{y}(t)$ is the rate of change in the firing rate, which is related to the level of dopamine stimulation $u(t)$ in patients, as

$$\ddot{y}(t) = -\dot{y}(t) + u(t)$$

.

a. Suppose $y(0) = 0, \dot{y}(0) = 0$, find the transfer function $G(s) = \frac{Y(s)}{U(s)}$.

b. What is $y(t)$ when there is a sudden increase in dopamine levels, i.e. $u(t) = 1_+(t)$?

c. A medical device company commissions you to design a controlled dopamine release device that stabilizes the amygdala in response to step changes in dopamine levels. Assume real time measurements of $y(t)$ are available. The device releases dopamine according to $u(t) = -e(t) + v(t)$, where $v(t)$ is a user set signal and $e(t) = K_1 y(t) + K_2 \dot{y}(t)$.

(a) Compute the transfer function, $C(s) = \frac{E(s)}{Y(s)}$

(b) Draw a block diagram showing the negative feedback circuit.

(c) Find the closed loop transfer function $G_{CL}(s) = Y(s)/V(s)$.

(d) Find the values of K_1 and K_2 so that the closed loop transfer function $G_{CL}(s)$ has poles at $s = -2, s = -3$.

d. In real life, instantaneous measurements of neural firing aren't possible. Suppose you could only know what $y(t)$ was after a delay τ, i.e. $y(t - \tau)$.

(a) Assuming τ is small, find the closed loop transfer function $G_{CL}(s)$ in terms of τ, K_1 and K_2.

(b) Suppose $\tau = 0.1$, find the values of K_1 and K_2 so that $G_{CL}(s)$ continues to have poles at $s = -2, s = -3$.