580.222 SYSTEMS AND CONTROLS

SPRING 2007- Homework Set 4
Distributed: April 11th 2007
Due: April 18th 2007

READING ASSIGNMENT
Chapter 4.1 until end of 4.1.1, 4.2 until beginning of 4.2.1, 4.3, 4.4.2 of Franklin

PROBLEMS TO BE GRADED

Problem 1:

3.38. Suppose that unity feedback is to be applied around the listed open-loop systems. Use Routh's stability criterion to determine whether the resulting closed-loop systems will be stable.

(a) $KG(s) = \frac{4(s+2)}{s(s^3+2s^2+3s+4)}$

(b) $KG(s) = \frac{2(s+4)}{s^2(s+1)}$

(c) $KG(s) = \frac{4(s^3+2s^2+s+1)}{s^2(s^3+2s^2-s-1)}$

Problem 2:

4.6. Consider a system with the plant transfer function $G(s) = 1/s(s+1)$. You wish to add a dynamic controller so that $\omega_n = 2$ rad/sec. and $\zeta \geq 0.5$. Several dynamic controllers have been proposed:

1. $D(s) = (s+2)/2$
2. $D(s) = 2\frac{s+2}{s+4}$
3. $D(s) = 5\frac{(s+2)}{s+10}$
4. $D(s) = 5\frac{(s+2)(s+0.1)}{(s+10)(s+0.01)}$

(a) Using MATLAB, compare the resulting transient and steady-state responses to reference step inputs for each controller choice. Which controller is best for the smallest rise time and smallest overshoot?

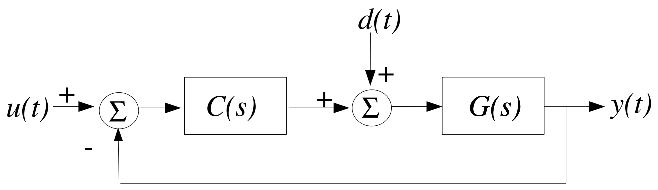
(b) Which system would have the smallest steady-state error to a ramp reference input?

(c) Compare each system for peak control effort; that is, measure the peak magnitude of the plant input $u(t)$ for a unit reference step input.

(d) Based on your results from parts (a) to (c), recommend a dynamic controller for the system from the four candidate designs.

Problem 3: *Guided robotic surgery*
Your company is in the process of developing a robotic arm that can be controlled remotely by surgeons. After much work, you have managed to design a system to track step changes in the input with overall transfer function $G(s) = \frac{1}{(s+3)(s+4)}$. However, in prototype testing, you notice that there is electrical noise from the circuit design being picked up by the output and translated into movement by the robot arm. You would like to design a controller so that the effects of the disturbance $d(t)$ will be removed from the output asymptotically.

Assume the noise is sinusoidal in nature, i.e. $d(t) = \sin(t)$. Assume the feedback architecture below. Design $C(s)$ and verify that $y(t)$ goes to 1 as t goes to ∞, in spite of $d(t)$.



Hint: Try $C(s) = \frac{(s+1)(s+2)}{D_c(s)}$ *and find what* $D_c(s)$ *should be.*

Problem 4:

4.35. The DC motor speed control shown in Fig. 4.51 is described by the differential equation

$$\dot{y} + 60y = 600v_a - 1500w,$$

where y is the motor speed, v_a is the armature voltage, and w is the load torque. Assume that the armature voltage is computed by using the PI control law

$$v_a = -\left(k_p e + k_I \int_0^t e\,dt\right),$$

where $e = r - y$.

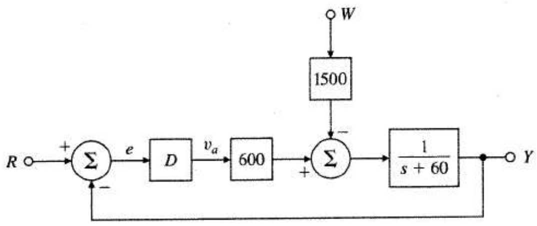


Figure 4.51
DC motor speed control block diagram for Problem 4.35

(a) Compute the transfer function from W to Y as a function of k_p and k_I.

(b) Compute values for k_p and k_I so that the characteristic equation of the closed-loop system will have roots at $-60 \pm 60j$.

Problem 5:

4.8. For the system in Problem 4.35, compute the following steady-state errors:

(a) for a unit-step reference input;

(b) for a unit-ramp reference input;

(c) for a unit-step disturbance input;

(d) for a unit-ramp disturbance input.

(e) Verify your answers to parts (a) to (d) using MATLAB. Note that a ramp response can be generated as the step response of a system modified by an added integrator at the reference input.