580.222 SYSTEMS AND CONTROLS

SPRING 2007- Controls Homework Set 5
Distributed: April 18th 2007
Due: April 25th 2007

READING ASSIGNMENT

Chapter 7.1-7.4 of Franklin
Study the following MATLAB commands: impulse, step, eig, pole, zero, tf, ss, tf2ss, zpk, conv, feedback, rlocus, plot, canon, ctrb, obsv

Problem 1: Franklin 4.28

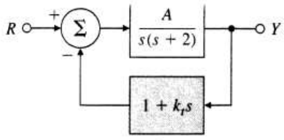
4.28. The feedback control system shown in Fig. 4.47 is to be designed to satisfy the following specifications: (1) steady-state error of less than 10% to a ramp reference input, (2) maximum overshoot for a unit-step input of less than 5%, and (3) 1% settling time of less than 3 sec.

(a) Compute the closed-loop transfer function.

(b) Sketch the region in the complex plane where the closed-loop poles may lie.

(c) What does specification (1) imply about the possible values of A?

Figure 4.47
Control system for Problem 4.28



(d) What does specification (3) imply about the closed-loop poles?

(e) Find the error due to a unit-ramp input in terms of A and k_t.

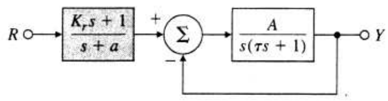
(f) Suppose $A = 32$. Find the value of k_t that yields closed-loop poles on the right-hand boundary of the feasible region. Use MATLAB to check whether this choice for k_t satisfies the desired specifications. If not, adjust k_t until it does.

(g) Using $A = 32$ and the value for k_t computed in part (f), estimate the settling time of the system. Use MATLAB to check your answer.

Problem 2: Franklin 4.36

4.36. Consider the system shown in Fig. 4.52, which consists of a prefilter and a unity feedback system.

Figure 4.52
Unity feedback system for Problem 4.36



(a) Determine the transfer function from R to Y.

(b) Determine the steady-state error due to a step input.

(c) Discuss the effect of different values of (K_r, a) on the system's response.

(d) For each of the three cases,

$$(1)\ A = 1, \tau = 1, \quad (2)\ A = 10, \tau = 1, \quad (3)\ A = 1, \tau = 2,$$

use MATLAB to find values for K_r and a so that (if possible)

i. the rise time is less than 1.5 sec,

ii. the overshoot is less than 20%,

iii. the settling time is less than 10 sec, and

iv. the steady-state error is less than 5%.

In cases in which the specifications are easily met, try to make the rise time as small as possible. If the specifications cannot be met, find the design to meet as many of the specifications as possible, in the order given.

Problem 3: Franklin 4.37

37. Consider the satellite-attitude control problem shown in Fig. 4.53 where the normalized parameters are

$J = 10$ spacecraft inertia, N-m-sec$^2/rad$

$\theta_r =$ reference satellite attitude, rad.

$\theta =$ actual satellite attitude, rad.

$H_y = 1$ sensor scale, factor volts/rad.

$H_r = 1$ reference sensor scale factor, volts/rad.

$w =$ disturbance torque, N-m

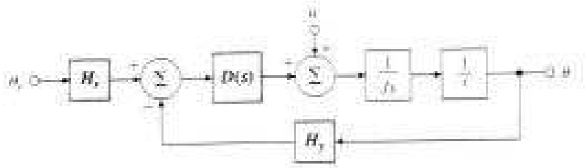


Figure 4.53: Satellite attitude control

(a) Use proportional control, **P**, with $D(s) = k_p$, and give the range of values for k_p for which the system will be stable.

(b) Use **PD** control and let $D(s) = (k_p + k_D s)$ and determine the system type and error constant with respect to reference inputs.

(c) Use **PD** control, let $D(s) = (k_p + k_D s)$ and determine the system type and error constant with respect to disturbance inputs.

(d) Use **PI** control, let $D(s) = (k_p + k_I/s)$, and determine the system type and error constant with respect to reference inputs.

(e) Use **PI** control, let $D(s) = (k_p + k_I/s)$, and determine the system type and error constant with respect to disturbance inputs.

(f) Use **PID** control, let $D(s) = (k_p + k_I/s + k_D s)$ and determine the system type and error constant with respect to reference inputs.

(g) Use **PID** control, let $D(s) = (k_p + k_I/s + k_D s)$ and determine the system type and error constant with respect to disturbance inputs.

Problem 4:

Consider a system G(s) = 1/((s+1)(s+2)(s+3)).

a. Compute the gain of a proportional controller that makes the system critically stable. Compute also the associated critical gain.

b. Find the parameters of a P, PI, and PID controller using the critical gain method of Ziegler and Nichols.

c. Implement each one of the three controllers in SIMULINK, and plot the response of each one of the closed loop systems to a step function. Compare the responses in terms of rising time, settling time, overshoot and zero steady state error.

d. Increase and reduce each of the gains of the PID controllers and plot the responses. Comment on the effect of each one of the gains.

e. Explain what the following MATLAB program does

```
% Part 1
num = 1
den = conv(conv([1 1],[1 2]),[1 3])
sys = tf(num,den)
pole(sys)
zero(sys)

% Part 2
canon(sys)
[A,B,C,D] = tf2ss(num,den)
```

```
eig(A)
ctrb(A,B)
ctrb(sys)

% Part 3
figure(1), step(sys)
figure(2), rlocus(sys)
feedback(sys,1)
pole(feedback(sys,60))
figure(3), step(feedback(sys,30))
```

Problem 5: *Controlled insulin release*
A biomedical device company has commissioned you to design a device that releases insulin in a controlled manner. You have a simple linear model of the device dynamics:

$$\dot{x_1} = x_2 \qquad \dot{x_2} = -x_1 - 2x_2 + u(t) + d(t)$$

where x_1 and x_2 is the amount and rate of insulin release respectively, $u(t)$ is the plant input and $d(t)$ is a disturbance input. The desired output $y(t)$ is the rate of insulin release, $x_2(t)$. Using the feedback architecture from class, you would like to design a controller $C(s)$ that tracks an input reference signal $r(t)$ with the following specifications: 1) $y(t)$ should respond to a unit step change in $r(t)$ with overshoot $M_p = 0.05$ and $t_{settling} = 0.1$. 2) The steady state error to a unit step or unit ramp disturbance should be < 0.1

a. Draw a block diagram of the feedback control system. Find the plant transfer function from the differential equations given.

b. Can we use a proportional controller to meet these specifications? Justify your answer.

c. Can we use a PI controller to meet these specifications? Justify your answer.