Given an image point in left image, what is the (corresponding) point in the right image, which is the projection of the same 3-D point.
Matching - Correspondence

Lambertian assumption

\[ I_1(x_1) = R(p) = I_2(x_2) \]

Rigid body motion

\[ x_2 = h(x_1) = \frac{1}{A_2(x)} (R_\lambda_1(X)x_1 + T) \]

Correspondence

\[ I_1(x_1) = I_2(h(x_1)) \]

Local Deformation Models

- Translational model

\[ h(x) = x + d \]

\[ I_1(x_1) = I_2(h(x_1)) \]

- Affine model

\[ h(x) = Ax + d \]

\[ I_1(x_1) = I_2(h(x_1)) \]

Transformation of the intensity values and occlusions

\[ I_1(x_1) = f_0(X, g)I_2(h(x_1)) + n(h(x_1)) \]
Feature Tracking and Optical Flow

- Translational model
  \[ I_1(x_1) = I_2(x_1 + \Delta x) \]
- Small baseline
  \[ I(x(t), t) = I(x(t) + ud_1, t + dt) \]
- RHS approx. by first two terms of Taylor series
  \[ \nabla I(x(t), t)^T u + I_t(x(t), t) = 0 \]
- Brightness constancy constraint

Aperture Problem

- Normal flow
  \[ u_{n_t} = \frac{\nabla I^T u}{\|\nabla I\|} \cdot \frac{\nabla I}{\|\nabla I\|} = -\frac{I_t}{\|\nabla I\|} \cdot \frac{\nabla I}{\|\nabla I\|} \]
Optical Flow

\[ E_b(u) = \sum_{(x,y)} W(x,y) [\nabla I'(x,y,t) \cdot u(x,y) + I_t(x,y,t)]^2 \]

- Integrate around over image patch

\[ \nabla E_b(u) = 2 \sum_{W(x,y)} \nabla I'(\nabla u + I_t) \]

\[ = 2 \sum_{W(x,y)} \left( \begin{bmatrix} I_x & I_y \\ I_x & I_y \end{bmatrix} u + \begin{bmatrix} I_xI_t \\ I_yI_t \end{bmatrix} \right) \]

\[ \begin{bmatrix} \sum I_x^2 & \sum I_xI_y \\ \sum I_xI_y & \sum I_y^2 \end{bmatrix} u + \begin{bmatrix} \sum I_xI_t \\ \sum I_yI_t \end{bmatrix} = 0 \]

\[ Gu + b = 0 \]

Optical Flow, Feature Tracking

\[ u = -G^{-1}b \]

\[ G = \begin{bmatrix} \sum I_x^2 & \sum I_xI_y \\ \sum I_xI_y & \sum I_y^2 \end{bmatrix} \]

Conceptually:
- rank(G) = 0 blank wall problem
- rank(G) = 1 aperture problem
- rank(G) = 2 enough texture - good feature candidates

In reality: choice of threshold is involved
Optical Flow

- Previous method - assumption locally constant flow

- Alternative regularization techniques (locally smooth flow fields, integration along contours)
- Qualitative properties of the motion fields

Feature Tracking
3D Reconstruction - Preview

• Compute eigenvalues of $G$
• If smallest eigenvalue $\sigma$ of $G$ is bigger than $\tau$ - mark pixel as candidate feature point

Point Feature Extraction

$$G = \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix}$$

• Compute eigenvalues of $G$
• If smallest eigenvalue $\sigma$ of $G$ is bigger than $\tau$ - mark pixel as candidate feature point

• Alternatively feature quality function (Harris Corner Detector)

$$C(G') = \det(G) + k \cdot \text{trace}^2(G)$$
Harris Corner Detector - Example

Wide Baseline Matching
Region based Similarity Metric

- Sum of squared differences
  \[ SSD(h) = \sum_{\tilde{x} \in W(x)} \| I_1(\tilde{x}) - I_2(h(\tilde{x})) \|^2 \]

- Normalize cross-correlation
  \[ NCC(h) = \frac{\sum_{W(x)} (I_1(\tilde{x}) - \bar{I}) (I_2(h(\tilde{x})) - \bar{I}_2)}{\sqrt{\sum_{W(x)} (I_1(\tilde{x}) - \bar{I})^2 \sum_{W(x)} (I_2(h(\tilde{x})) - \bar{I}_2)^2}} \]

- Sum of absolute differences
  \[ SAD(h) = \sum_{\tilde{x} \in W(x)} | I_1(\tilde{x}) - I_2(h(\tilde{x})) | \]

Edge Detection

- Canny edge detector
  - Compute image derivatives
  - if gradient magnitude > \( \tau \) and the value is a local maximum along gradient direction – pixel is an edge candidate
Line fitting

- Edge detection, non-maximum suppression
  (traditionally Hough Transform - issues of resolution, threshold selection and search for peaks in Hough space)
- Connected components on edge pixels with similar orientation
  - group pixels with common orientation

Non-max suppressed gradient magnitude

Line Fitting

\[
A = \begin{bmatrix}
\sum x_i^2 & \sum x_i y_i \\
\sum x_i y_i & \sum y_i^2
\end{bmatrix}
\]

second moment matrix associated with each connected component

- Line fitting Lines determined from eigenvalues and eigenvectors of A
- Candidate line segments - associated line quality